Introduction - Bayesian Statistics

Bayesian inference

Bayesian inference models the process of learning. That is, we start with a certain level of belief, however vague, and through the accumulation of experience, our belief becomes more fine-tuned. Some people take a dislike to Bayesian inference because it is overtly subjective and they like to think of statistics as being objective. We don't agree, because any statistical analysis is necessarily subjective as a result of the need to make assumptions, but also because many approximations are accepted without question, even without warning that they have been made. For that reason we appreciate the extra transparency of Bayesian inference, but it also frequently provides answers where classical statistics cannot. Perhaps more importantly for us as risk analysts, Bayesian inference encourages our clients to think about the level of knowledge *they* have about their problem, and what that means to them.

Bayesian inference is an extremely powerful technique, based on Bayes' Theorem (sometimes called Bayes' Formula), for using data to improve one's estimate of a parameter. There are essentially three steps involved:

1. Constructing a confidence distribution of the parameter before analyzing the new data set. This is called the prior distribution;
2. Find an appropriate likelihood function for the observed data; and
3. Modify the prior distribution using the likelihood function to get a revised estimate known as the posterior distribution.

Bayesian inference options

Number of parameters to estimate	1	2	3+
Prior selection:			
	Simple	Uncorrelated	
		Correlated	
Analytical methods:	Markov Chain Monte Carlo		
	Accept / Reject Simulation		
	Construction		
	Conjugate		

This section starts with some introductory topics:

Introduction to the concept and some simple examples

How to determine prior distributions

How to determine likelihood functions

We then turn to the actual execution of a Bayesian inference for risk analysis models with Crystal Ball by looking at various techniques for arriving at the posterior distribution:

Construction method

Conjugate prior method

Most important of all, we offer a number of worked examples:

Examples of Bayesian inference calculations

General estimation problems

A simple Bayesian inference example using construction

Simple construction model showing the interaction between likelihood functions and informed priors

A simple construction model illustrating the importance of the prior distribution

A model to show how to incorporate hyperparameters by simulation, as well as offering both simulation and construction approaches to determining the posterior distribution

A fun model of a classic problem showing the sometimes non-intuitive nature of the Bayesian result

A bit gross, but this example shows how you can perform Bayesian inference calculations with very involved stochastic processes (i. e. very complicated likelihood functions) by using Monte Carol simulation with accept/reject criteria.

A simple construction example that shows how we use data that describe being above or below a threshold, instead of exact observations

Taylor series approximation to a Bayesian posterior distribution

Showing how Taylor series expansion lets you determine the normal approximation to posterior distributions, and a method for algebraically obtaining the standard deviation

Example of a Taylor series expansion

Two common statistical problems

A standard statistics problem with the same outcome as the classical method

Another standard statistics problem with the same outcome as the classical method

Once you have reviewed the material in this section, you might like to test how much you have learned by taking the self-test quiz:

A quiz on Bayesian inference: Take quiz