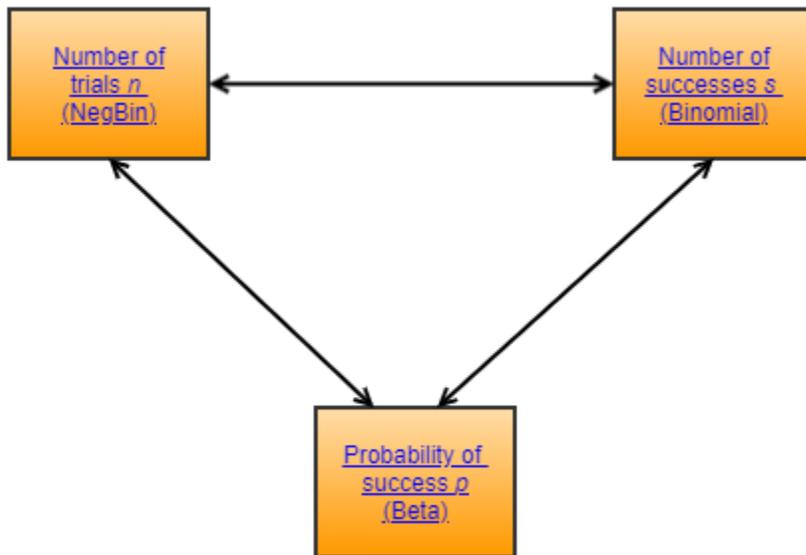# Introduction - The binomial process

The binomial process

Definition

A binomial process is a random counting system where there are n independent identical trials, each one of which has the same probability of success p, which produces s successes from those n trials (where $0 \le s \le n$ and $n > 0$ obviously). There are thus three parameters $\{n, p, s\}$ that between them completely describe a binomial process. Associated with each of these three parameters are three distributions that describe the uncertainty about or variability of these parameters. The three distributions require that one has knowledge of two parameters in order to use these distributions to estimate the third.

Simple example

The simplest example of a binomial process is the toss of a coin. If we define "heads" as a success, each toss has the same probability of success p (0.5 for a fair coin). Then, for a given number of trials n (tosses of a coin), the number of successes will be s (the number of "heads". Each trial can be thought of as a random variable that returns either a 1 with probability p or a 0 with probability $(1-p)$. Such a trial is often known as a *Bernoulli* trial, and the probability $(1-p)$ is often given the label q.

The distributions

As explained above, there are essentially three distributions that describe the uncertainty or randomness of the three components of the binomial process, s, n and p. These three distributions are described in the following sections. The relationship between these distributions is also shown graphically in the section Introduction to stochastic processes. Finally, whereas the second section describes the randomness of the number of trials n to achieve s successes, the fourth section describes a distribution of the uncertainty (not randomness !) of n, after observing s successes. The answers can be different!

1. Distribution of the number of successes s in n trials, each with probability p
2. Distribution of the number of trials n needed to obtain s successes, each with probability p
3. Estimation of the probability p after having observed s successes in n trials
4. Estimation of the number of trials n made after having observed s successes with probability p

Once you have reviewed the material in this section, you might like to test how much you have learned by taking the self-test quiz:

A quiz on The binomial process: **Take quiz**

Summary of results for the binomial process

Quantity	Formula	Notes
Number of successes	s = Binomial(p,n)	
Probability of success	p = Beta(s+1,n-s+1,1)	assuming Uniform prior
	p = Beta(a+s, b+n-s, 1)	assuming Beta($a,b,1$) prior
Number of trials	n = NegBinomial(p,s)	when last trial is a success
	n = NegBinomial(p,s+1)-1	when last trial not known to be a success

Useful Excel functions:

Use	Function	Explanation
Binomial probability	=BINOMDIST(x,n,p, cumulative)	The Binomial probability of: s = x (cumulative = FALSE), or: s <= x (cumulative = TRUE)
Negative Binomial probability	=NEGBINOMDIST(x,s,p)	The Negative Binomial probability of getting exactly x failures
Beta probability	=BETADIST(x,a1,a2)	The Beta probability of getting <= x
Beta inverse probability	=BETAINV(P,a1,a2)	The value x such that P("true prev' <= x) = P
Combinations	=COMBIN(n,x)	The binomial coefficient, i.e. the number of different orderings one can get in choosing x items from n
Beta probability density	There is no Excel density function, but we can use the formula: =x^(a1-1)*(1-x)^(a2-1)/EXP(GAMMALN(a1)+GAMMALN(a2)-GAMMALN(a1+a2))	