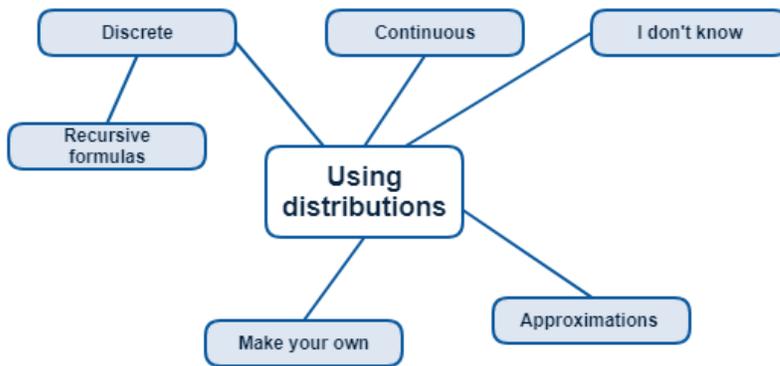


# Distributions

## Use of Distributions

Distributions are used in risk analysis to model three conceptually different things:

1. The variability of individuals in a population (frequency distribution);
2. The value of a random variable (probability distribution); and
3. The uncertainty we have about a fixed, but imprecisely known, parameter in nature (uncertainty distribution).



This section is a resource to allow you to look up specific distributions by name, to see how they are used, learn how to generate their values when they are not directly available in your preferred software, and review the most useful equations associated with each distribution. We have also described the inter-relationships between various distributions and provided links to related topics.

The distributions are split into two categories: [discrete](#) and [continuous](#). We explain what you need to think about when [selecting a distribution](#) to go in your model. We also provide a section that describes how you can [approximate one distribution with another](#), or use [recursive formulas](#), so that you can avoid the parameter restrictions set by your simulation software.

Finally, if none of the distributions we describe suit your purpose, you can always [create your own](#)! We show you a variety of ways to do that.

Once you have reviewed the material in this section, you might like to test how much you have learned by taking the self-test quizzes:

Take

A quiz on Distributions: [quiz](#)

Take

A quiz on Probability distribution identities (intermediate level): [quiz](#)

Take

A quiz on Probability distribution identities (advanced level): [quiz](#)

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## Useful formulas and functions

When reviewing the formulae associated with each distribution you will frequently come across a number of unusual (though not very complicated!) mathematical functions that are worth explaining here:

### Binomial coefficient

The binomial coefficient is defined as follows:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

where:  $n! = 1 * 2 * 3 * \dots * (n-1) * n$

In Excel use =COMBIN( $n,r$ ). The Excel function FACT( $n$ ) also returns  $n!$

### The Gamma function

$\Gamma(n)$  is the gamma function and has the following properties:

$$\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt$$

$\Gamma(n+1) = n\Gamma(n) = n!$  where  $n$  is an integer

$\Gamma(0.5) =$

$\Gamma(0) = 1$

So, for example,  $\Gamma(1.5) = 1/2 * \Gamma(0.5) = 1/2$ . Excel offers the function GAMMALN( ) which returns the natural log of the Gamma function, so to get  $\Gamma(n)$  you write =EXP(GAMMALN( $n$ )).

Other functions

<b>tan-1(x)</b>	The arc tan of x in radians. =ATAN(x) in Excel
<b> x </b>	The absolute value of x, i.e.  -x  = x,  x  = x. Use =ABS(x) in Excel
<b>csc(x)</b>	The cosecant of x, = 1/sin(x). Use =1/SIN(x) in Excel

$\lfloor x \rfloor$	The nearest integer at or below $x$ . Use =ROUNDDOWN( $x,0$ ) in Excel
$\lceil x \rceil$	The nearest integer at or above $x$ . Use =ROUNDUP( $x,0$ ) in Excel
$\ln(x)$	The natural log of $x$ , so that $x = \exp(\ln(x))$ . Use =LN( $x$ ) in Excel
$\exp(x)$ , <b>ex</b>	The natural exponent of $x$ , $= 2.718281828\dots x$ . Use =EXP( $x$ ) in Excel

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