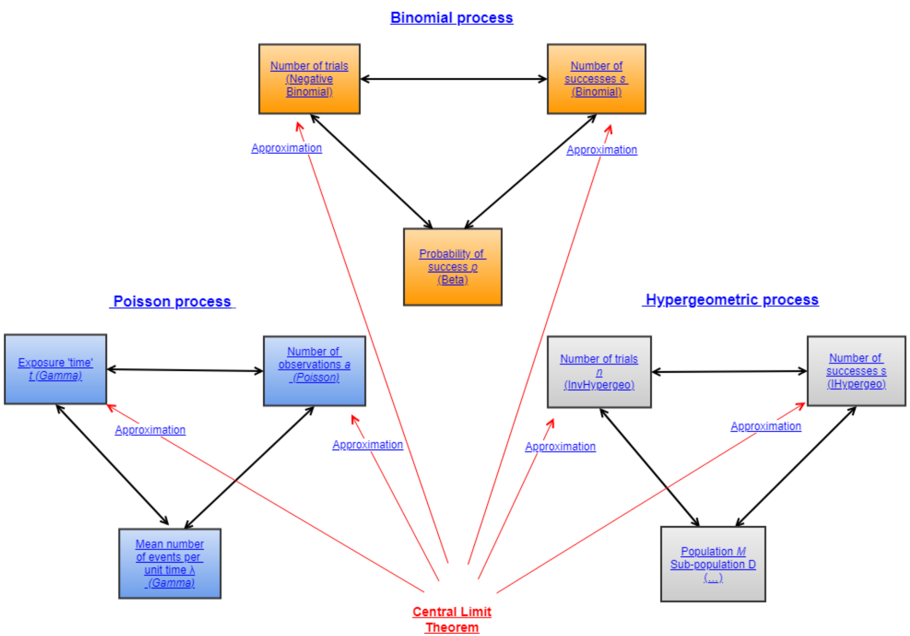
# Introduction - Stochastic processes

## Stochastic Processes

A stochastic process is a system of countable events, where the events occur according to some well-defined random process. Strictly speaking, a stochastic process is also concerned with the sequence in which the events occur in time, but we shall take the more usual broader definition to include counting systems where the order is of no importance. This section describes four very fundamental stochastic processes: the Binomial, Poisson, Hypergeometric, and Central Limit Theorem (CLT).

Click the boxes below to explore the main **distributions** of the four fundamental stochastic processes, and on the **arrows to see how they relate**. Note that the **top half of each box explains the parameter and derivations**, and the **bottom half describes the distribution(s)** used to model each parameter




## Some motivation

Having an understanding of stochastic processes is really quite critical to good risk analysis modeling, so we encourage you to put some effort into this section. In school physics lessons we learn about different types of motion (simple harmonic, rolling, sliding, parabolic flight, etc). Attached to each type of motion we learn some equations that completely describe whatever we wish to know. Then we go out into the world and see any number of examples of these types of motion. Learn to identify the types of motion in the real world, and know the associated equations, and you can describe a huge number of problems, from planetary motion, electrons around an atom, missile and rocket trajectories, to rolling a snowball down a hill. The same applies to these stochastic processes: understand their characteristics and thus how to spot them in the real world, learn how to use the equations, and you will be able to solve a vast number of problems with ease.

## The relationship between stochastic processes

The diagram above illustrates how these stochastic processes are related. For example, the binomial process has three parameters: n – the number of trials to be run, s – the number of successes that may result, and p – the probability that a trial will be a success. If we know any two, we can estimate the third, as depicted by the arrows joining these three components together. Thus, for example, the Beta, Binomial and Negative Binomial distributions are describing different aspects of the same process and are therefore intimately related.

More interestingly still, the Poisson process can be viewed very similarly and since the Poisson mathematics are derived from the Binomial process where n is made large and p is small, there is a strong relationship between the corresponding Poisson and Binomial distributions. So, for example, a Binomial(p, n) distribution for s will look progressively more like a Poisson(n*p) as n gets larger and p gets smaller. The Hypergeometric process is similarly linked.

Central Limit Theorem applies to many of these distributions and explains why they look very similar to Normal distributions for certain parameter values.

A very great deal of risk analysis problems can be tackled with a good knowledge of these four processes. We look at the theory and assumptions behind each process, and the distributions that are used in their modeling. This approach provides us with an excellent opportunity to become very familiar with a number of important distributions, and to see the relationships between them, even between the distributions of the different stochastic processes.

## Extra

This section also discusses a generalization of the Poisson process where the times between events are independent and identically distributed with an arbitrary distribution, a type of randomness known as a renewal process which is often used in modeling equipment reliability, for example.

Finally, some examples are given of mixture processes. These are random processes where one or more of the defining parameters (like a binomial probability, for example) may itself be a random variable. There are some very useful theoretical results that come out of mixture processes, and in Monte Carlo simulation this is something that do we quite naturally anyway by simply nesting distributions.